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Bulk Viscous Bianchi Type I Cosmological Models with Time-Dependent Cosmological Term Λ

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Abstract Bianchi type I cosmological models with time-varying cosmological constant Λ and bulk viscous fluid are investigated. Cosmic matter is chosen to obey a barotropic equation of state. Exact solutions of Einstein's field equations are obtained assuming the volume expansion θ proportional to the eigen values of shear tensor σ_{ij}. Physical and kinematical properties of the models are discussed considering bulk viscosity to be a power function of matter density.

Keywords Bianchi I · Bulk viscous · Cosmological constant · Time dependent

1 Introduction

The cosmological constant problem is one of the outstanding problems in cosmology [1, 2]. A wide range of observations suggest that universe possesses a non-zero cosmological constant [3]. In modern cosmological theories, the cosmological constant remains a focal point of interest. A cosmological term corresponds to the energy-density of vacuum in the context of quantum field theory. The cosmological term which is a measure of the energy of empty space, provides a repulsive force opposing the gravitational pull between the galaxies. The energy it represents, counts as mass by Einstein's mass-energy equivalent formula. But recent researches suggest that the cosmological term has a very small value of the order of 10^{-58} cm^{-2} [4]. Linde [5] has suggested that Λ is a function of temperature and is related to the spontaneous symmetry breaking process. Therefore, it could be a function of time in a spatially homogeneous expanding universe [6]. The latest measurements of the Hubble parameter [7, 8] point to an intrinsic fragility of the standard (Photon conserving) FRW cosmology in such a way that models without a cosmological constant seem to be effectively

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ruled out [9, 10]. In the determination of Hubble's constant (H_0), it is difficult for the FRW models without a cosmological constant to lead to the age of the universe greater than that of stars [10–12].

The observed physical phenomena such as the large entropy per baryon and the remarkable degree of isotropy of the cosmic background radiation, suggest dissipative effects analysis in cosmology. There are several processes which give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era. Bulk viscosity is also associated with the GUT phase transition and string creation. It has been argued for a long time that the dissipative process in the early stages of cosmic expansion may well account for the high degree of isotropy we observe today. Dissipative effects including both the bulk and shear viscosities, play a very significant role in the early evolution of the universe. From a physical point of view, the inclusion of dissipative terms in the energy-momentum tensor of the cosmological fluid, seems to be the best motivated generalization of the matter term of the gravitational field equations. To study the effect of bulk viscosity, the first relativistic theory of non-equilibrium thermodynamics, was developed by Eckart [13]. Landau and Lifshitz [14] have also presented essentially equivalent formulations. Padmanabhan and Chitre [15] have investigated that presence of bulk viscosity leads to inflationary like solutions in general relativity. Another peculiar characteristic of bulk viscosity is that it acts like a negative energy field in an expanding universe [16]. Pavon et al. [17] have studied the evolution of FRW universe with a flat spatial section filled by viscous fluid where coefficient of bulk viscosity obeys a power law relation with energy density. Romano and Pavon [18] have investigated the evolution of Bianchi Type I universe with viscous fluid. Mak and Harko [19] have studied the dynamics of a causal bulk viscous fluid cosmological model with constantly decelerating Bianchi Type I space-time. Saha [20–22] in a series of papers discussed Bianchi Type I universe with viscous fluid. The effect of bulk viscosity on the cosmological evolution has been investigated by number of authors viz. Pradhan et al. [23–25], Singh et al. [26], Sahni and Starobinsky [27], Padmanabhan [28], Peebles [29]. Recently Bali and Pradhan [30] have investigated Bianchi Type III string cosmological model with time- dependent bulk viscosity.

2 The Field Equations and General Discussion

We consider the Bianchi type I space–time in orthogonal form represented by

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2 \tag{1}$$

We assume that the cosmic matter consists of bulk viscous fluid given by the energy momentum tensor.

$$T_i^j = (\rho + \bar{p})v_i v^j + \bar{p} g_i^j \tag{2}$$

where ρ is the energy density of matter, $\bar{p}$, the effective pressure and v_i, the four–velocity of the fluid satisfying the relation

$$v_i v^i = -1 \tag{3}$$

The effective pressure $\bar{p}$ is related to the equilibrium pressure p by

$$\bar{p} = p - \zeta v^i_{;i} \tag{4}$$

where ζ stands for the coefficient of bulk viscosity that determines the magnitude of the viscous stress relative to expansion. We shall use non-causal theory to study the dissipative mechanism. The Einstein field equations (in gravitational units $8\pi G = c = 1$) with time-dependent-cosmological term $\Lambda(t)$ are

$$R_i^j - \frac{1}{2}Rg_i^j = -(T_i^j - \Lambda g_i^j) \tag{5}$$

In co-moving system of coordinates, the field equations (5) for the metric (1) and matter distribution (2) yield.

$$\bar{p} - \Lambda = -\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} \tag{6}$$

$$\bar{p} - \Lambda = -\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC} \tag{7}$$

$$\bar{p} - \Lambda = -\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} \tag{8}$$

$$\rho + \Lambda = \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} \tag{9}$$

where an overdot means ordinary differentiation with respect to cosmic time t. In view of vanishing divergence of Einstein tensor, we obtain

$$\dot{\rho} + (\rho + \bar{p})\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{C} + \frac{\dot{C}}{C}\right) + \dot{\Lambda} = 0 \tag{10}$$

When Λ is constant, we get the continuity equation for matter. In view of energy conservation, (10) shows that a decaying vacuum term Λ transfers energy continuously to the material component.

The effective time–dependent cosmological term is regarded as second fluid component with energy density $\rho_v = \Lambda(t)$, where ρ_v is vacuum density. On thermo-dynamical grounds bulk viscosity coefficient ζ is positive, assuring that the viscosity pushes the effective pressure towards negative values. As usual, we consider that the non-vacuum component of matter obeys the equation of state

$$p = \omega\rho \tag{11}$$

where $\omega \in [0, 1]$ specifies if the fluid component is radiation ($\omega = 1/3$) or dust ($\omega = 0$).

Let R be the average scale factor of Bianchi type I universe i.e.

$$R^3 = \sqrt{-g} = ABC \tag{12}$$

From (6), (7) and (8), we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{R^3} \tag{13}$$

and

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{R^3} \tag{14}$$

where k_1, k_2 are constants of integration.

On integration (13) and (14) give

$$A = m_1 R \exp\left[\frac{(2k_1+k_2)}{3}\int \frac{dt}{R^3}\right]$$
$$B = m_2 R \exp\left[\frac{(k_2-k_1)}{3}\int \frac{dt}{R^3}\right] \tag{15}$$
$$C = m_3 R \exp\left[-\frac{(k_1+2k_2)}{3}\int \frac{dt}{R^3}\right]$$

where m_1, m_2, m_3 are arbitrary constants of integration satisfying

$$m_1 m_2 m_3 = 1$$

We introduce volume expansion θ and shear σ as usual

$$\theta = v^i_{;i}, \qquad \sigma^2 = \frac{1}{2}(\sigma_{ij}\sigma^{ij}) \tag{16}$$

σ_{ij} being shear tensor.

For the Bianchi Type I metric expressions for the dynamical scalars come out to be

$$\theta = \frac{3\dot{R}}{R}, \qquad \sigma = \frac{k}{\sqrt{3}R^3}, \qquad k^2 = k_1^2 + k_2^2 + k_1 k_2 \tag{17}$$

In analogy with a FRW universe, we define a generalized Hubble parameter H and the deceleration parameter q as

$$H = \frac{\dot{R}}{R} \tag{18}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -\frac{\dot{H}}{H^2} - 1 \tag{19}$$

Equations (6)–(9) can be written in terms of H, σ and q as

$$\bar{p} - \Lambda = H^2(2q-1) - \sigma^2 \tag{20}$$

$$\rho + \Lambda = 3H^2 - \sigma^2 \tag{21}$$

Equation (21) implies that

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{\rho}{\theta^2} - \frac{\Lambda}{\theta^2}$$

Therefore, $0 \le \frac{\sigma^2}{\theta^2} \le \frac{1}{3}$ and $0 \le \frac{\rho}{\theta^2} \le \frac{1}{3}$ for $\Lambda \ge 0$.

Thus, the presence of a positive Λ puts restriction on the upper limit of anisotropy whereas a negative Λ provides more room for the anisotropy. From (20), we obtain

$$\frac{d\theta}{dt} = \frac{-3}{2}\left\{p + \frac{\theta^2}{3} + \sigma^2 - \zeta\theta - \Lambda\right\} \tag{22}$$

Thus, a positive Λ will arrest the rate of decrease of volume expansion and a negative Λ would promote it. Also the presence of viscosity will slow down the rate of decrease. It is also to mention that the rate of decrease of volume expansion is faster in anisotropic background in comparison to that in an isotropic one. Also, (20) and (21) give

$$\frac{\Lambda}{H^2} = 2 - q + \frac{(\omega - 1)\rho}{2H^2} - \frac{3\zeta}{2H} \tag{23}$$

If $q < 2 + \frac{(\omega-1)\rho}{2H^2} - \frac{3\zeta}{2H}$, $\Lambda > 0$ whereas $\Lambda < 0$ for $q > 2 + \frac{(\omega-1)\rho}{2H^2} - \frac{3\zeta}{2H}$. Also, $\dot{\sigma} = -3\sigma H$ implying that σ decreases in an evolving universe and for infinitely large values of R, σ becomes negligible.

It is to note that energy density of the universe is a positive quantity. It is believed that at the early stages of the evolution when the average scale factor R was close to zero, the energy density of the universe was infinitely large. On the other hand, with the expansion of the universe i.e. with increase of R, energy density decreases and an infinitely large R corresponds to a ρ close to zero. In that case from (21), we obtain

$$3H^2 - \Lambda \to 0. \tag{24}$$

From (24) one concludes that:

(i) Λ is essentially non-negative.
(ii) in the absence of a Λ-term beginning from some value of R, the evolution of the universe becomes stand-still i.e. R becomes constant since H becomes zero.
(iii) in case of a positive Λ, the process of evolution of the universe never comes to a halt. Moreover, it is believed that the presence of dark energy (given by positive Λ) results in accelerated expansion of the universe. As far as negative Λ is concerned, its presence imposes some restriction on ρ i.e. ρ can never be small enough to be ignored. It means, in that case there exists some upper limit for R as well. It is worth mentioning here that Saha [31] has also given such a conclusion in his paper.

From (10), we obtain

$$\frac{1}{R^{3(\omega+1)}} \frac{d}{dt}(\rho R^{3(\omega+1)}) = 9\zeta H^2 - \dot{\Lambda} \tag{25}$$

Thus, decaying vacuum energy and viscosity of the fluid lead to the matter production. Using (15), we obtain the non-vanishing physical components of conformal curvature tensor for the metric (1) as:

$$C_{(1212)} = (k_1 + 2k_2)\frac{H}{3R^3} + \frac{(-k_1^2 + 2k_2^2 + 2k_1k_2)}{9R^6} \tag{26}$$

$$C_{(1313)} = (k_1 - k_2)\frac{H}{3R^3} + \frac{(-k_1^2 - k_2^2 - 4k_1k_2)}{9R^6} \tag{27}$$

$$C_{(2323)} = (-2k_1 - k_2)\frac{H}{3R^3} + \frac{(2k_1^2 - k_2^2 + 2k_1k_2)}{9R^6} \tag{28}$$

We observe that for a decelerating universe, the model becomes conformally flat for large values of R.

3 Solution of the Field Equations

In order to solve (13) and (14) involving three unknowns A, B and C, we require one more condition. It is believed that evolution of one parameter should also be responsible for the evolution of the others [32]. We assume that the volume expansion θ is proportional to eigen values of the shear tensor σ_{ij}. Following Roy et al. [33], we take the volume expansion θ having a constant ratio to the anisotropy in the direction of unit space like vector λ^i i.e. $\frac{\theta}{\sigma_{ij}\lambda^i\lambda^j}$ is constant. In general, the above condition gives rise to

$$A = B^m C^n \tag{29}$$

where m and n are constants.

Using (29) in (13) and (14) we obtain

$$\begin{aligned} C &= b_1(k_3 t + k_4)^{\frac{k_1-(m-1)k_2}{(m+n+2)k_1-(m-2n-1)k_2}} \\ B &= b_2(k_3 t + k_4)^{\frac{k_1+k_2 n}{(m+n+2)k_1-(m-2n-1)k_2}} \quad \text{for } \frac{k_1}{k_2} \neq \frac{m-2n-1}{m+n+2} \end{aligned} \tag{30}$$

$$\begin{aligned} C &= k_5 \exp\left[\frac{-k_2(m+1)t}{m+n+2}\right] \\ B &= b_3 \exp\left[\frac{k_2(n+1)t}{m+n+2}\right] \quad \text{for } \frac{k_1}{k_2} = \frac{m-2n-1}{m+n+2} \end{aligned} \tag{31}$$

provided $m + n \neq 1$. In the above k_3, k_4, k_5 and b_1, b_2, b_3 are constants. For these solutions, metric (1) takes the following forms after suitable transformations:

$$\begin{aligned} ds^2 = {} & -dT^2 + T^{\frac{2(m+n)k_1+2nk_2}{(m+n+2)k_1-(m-2n+1)k_2}} dX^2 + T^{\frac{2k_1+2nk_2}{(m+n+2)k_1-(m-2n-1)k_2}} dY^2 \\ & + T^{\frac{2k_1-2(m-1)k_2}{(m+n+2)k_1-(m-2n+1)k_2}} dZ^2 \quad \text{for } \frac{k_1}{k_2} \neq \frac{m-2n-1}{m+n+2} \end{aligned} \tag{32}$$

$$\begin{aligned} ds^2 = {} & -dT^2 + \exp\left[\frac{2k_2(m-n)T}{m+n+2}\right] dX^2 + \exp\left[\frac{2k_2(n+1)T}{m+n+2}\right] dY^2 \\ & + \exp\left[-\frac{2k_2(m+1)T}{m+n+2}\right] dZ^2 \quad \text{for } \frac{k_1}{k_2} = \frac{m-2n-1}{m+n+2} \end{aligned} \tag{33}$$

For the model (32) physical parameters in terms of cosmic time T have the following expressions:

$$\bar{p} - \Lambda = \frac{1-k^2}{3T^2} \tag{34}$$

$$\rho + \Lambda = \frac{1-k^2}{3T^2} \tag{35}$$

$$\theta = \frac{1}{T} \tag{36}$$

$$\sigma = \frac{k}{\sqrt{3}T} \tag{37}$$

We observe that the universe starts from a singular state at $T = 0$. It begins expanding with a big bang at $T = 0$ and continues to expand till $T = \infty$. Since $\frac{\sigma}{\theta} = \frac{k}{\sqrt{3}}$, the model does not approach isotropy. However, if k is small, the model is quasi-isotropic i.e. $\sigma/\theta \approx 0$. Using (34), (35) and (11), we obtain

$$\rho = \frac{1}{(\omega+1)}\left\{\frac{2(1-k^2)}{3T^2} + \frac{\zeta}{T}\right\} \tag{38}$$

and

$$\Lambda = \frac{(1-k^2)(\omega-1)}{3(\omega+1)T^2} - \frac{\zeta}{(\omega+1)T} \tag{39}$$

Thus, we conclude that the presence of viscosity is to increase the value of matter density and to decrease the value of vacuum energy density. Also from (34) and (35), we obtain

$$(\rho - p) + \zeta\theta + 2\Lambda = 0. \tag{40}$$

Thus, for stiff matter distribution, cosmological term Λ is essentially zero in the absence of viscosity. The model (33) is not of much interest since it reduces to a static solution. The coefficient of bulk viscosity is assumed to be function of energy density ρ [34] or volume expansion θ:

$$\zeta = \zeta_0\rho^\alpha \quad \text{or} \quad \zeta = \zeta_0\theta^\beta$$

where ζ_0 (≥ 0), α (≥ 0) and β are constants. If $\alpha = 1$, we obtain radiating fluid whereas $\alpha = 3/2$ may correspond to a string dominated universe [35]. However, more realistic models [36] are based on α lying in the regime $0 \leq \alpha \leq \frac{1}{2}$. We discuss these physical assumptions in the following:

3.1 Model I ($\zeta = \zeta_0$)

When $\alpha = 0$, $\zeta = \zeta_0$. From (34) and (35), we get

$$\rho = \frac{1}{(\omega+1)}\left\{\frac{2(1-k^2)}{3T^2} + \frac{\zeta_0}{T}\right\}$$

and

$$\Lambda = \frac{(\omega-1)(1-k^2)}{3(\omega+1)T^2} - \frac{\zeta_0}{(\omega+1)T}$$

Thus, matter density ρ and vacuum density Λ are infinite at the initial singularity and they decrease to become zero at late times. We observe that ρ/θ^2 is constant at $T = 0$ and $\rho/\theta^2 \to \infty$ as $T \to \infty$. Thus ρ and θ^2 are comparable at the initial stage whereas matter density dominates the expansion at late times.

3.2 Model II ($\zeta = \zeta_0\rho$)

When $\alpha = 1$, we obtain $\zeta = \zeta_0\rho$ and (34) and (35) give

$$\rho = \frac{2(1-k^2)}{3T\{(\omega+1)T - \zeta_0\}}$$

and

$$\Lambda = \frac{(1-k^2)}{3T^2}\left\{\frac{(\omega-1)T - \zeta_0}{(\omega+1)T - \zeta_0}\right\}$$

In this case also, ρ and Λ are infinite at $T=0$ and become zero at $T=\infty$. Since $\rho/\theta^2=0$ at $T=0$ and $\rho/\theta^2 \to$ constant as $T\to\infty$, initial stage is dominated by the expansion whereas at late times ρ and θ^2 become comparable.

3.3 Model III ($\zeta=\zeta_0\rho^{1/2}$)

For $\alpha=\frac{1}{2}$, we obtain $\zeta=\zeta_0\rho^{1/2}$. Using (34) and (35), we get

$$\rho=\frac{\frac{6\zeta_0^2+8(\omega+1)(1-k^2)}{3T^2}\pm\frac{2\zeta_0^2}{T^2}\sqrt{1+\frac{8(\omega+1)(1-k^2)}{3\zeta_0^2}}}{4(\omega+1)^2}$$

$$\Lambda=\frac{(1-k^2)}{3T^2}-\frac{\frac{(6\zeta_0^2+8(\omega+1)(1-k^2)}{3T^2}\pm\frac{2\zeta_0^2}{T^2}\sqrt{1+\frac{8(\omega+1)(1-k^2)}{3\zeta_0^2}}}{4(\omega+1)^2}$$

In this case, we observe that matter energy density ρ and vacuum energy density Λ are infinite at the initial singularity and they become zero for large values of T. Since ρ/θ^2 is constant, ρ and θ^2 are comparable throughout the evolution.

3.4 Model IV ($\zeta\propto\theta$)

We assume $\beta=1$ i.e. $\zeta=\zeta_0\theta$. For this choice, we obtain

$$\rho=\frac{2(1-k^2)+3\zeta_0}{3(\omega+1)T^2}$$

and

$$\Lambda=\frac{(1-k^2)(3\omega+1)-3\zeta_0}{3(\omega+1)T^2}$$

In this case also matter density ρ and vacuum density Λ are infinite initially and they become zero for large values of T. ρ and θ^2 are comparable throughout the evolution.

3.5 Model V ($\zeta\propto 1/\theta$)

For $\beta=-1$, we get $\zeta=\zeta_0/\theta$. In this case ρ and Λ are given by

$$\rho=\frac{1}{(\omega+1)}\left\{\frac{2(1-k^2)}{3T^2}+\zeta_0\right\}$$

and

$$\Lambda=\frac{(1-k^2)(w-1)}{3(w+1)T^2}-\frac{\zeta_0}{w+1}$$

At $T=0$, ρ and Λ are infinite and for large values of T, they become finite. ρ and θ^2 are comparable at the initial singularity whereas matter density dominates the expansion at late times.

4 Conclusion

Evolution of Bianchi Type I cosmological models is studied in the presence of bulk viscous fluid source and time-dependent cosmological term Λ. The cosmic fluid is chosen to obey a barotropic equation of state. We have assumed the volume expansion θ proportional to the eigen values of shear tensor σ_{ij} which gives rise to a relation between the metric potentials A, B, C. We observe that all the matter and radiation is concentrated at the big bang epoch and cosmic expansion is driven by big bang impulse. The model universe (32) has singular origin at $T = 0$. The rate of expansion slows down and drops to zero as $T \to \infty$. The pressure and energy densities become negligible except in the case (3.5) where they become finite for large values of T. We find that the presence of bulk viscosity is to increase the value of matter density ρ and to decrease the value of vacuum energy density Λ. Since σ/θ is constant, the model does not approach isotropy. However, for small k, the model is quasi-isotropic i.e. $\sigma/\theta \approx 0$. We also obtain that cosmological term Λ is zero in the absence of bulk viscosity for stiff matter. In the absence of bulk viscosity, $\Lambda \propto T^{-2}$ which is considered as standard.

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